Math 312- HW1 Due Date: 5/10/17

1. Let a belong to a ring R. Let $S = \{x \in R \mid ax = 0\}$. Prove that S is a subring of R.

2. Let R be a ring. The center of R is the set $\{x \in R \mid ax = xa \text{ for all } a \in R\}$. Prove that the center of a ring is a subring.

3. Suppose that a and b belong to a commutative ring R. If a is a unit in R and $b^2 = 0$, prove that $a + b$ is a unit in R.

4. A Boolean ring R is a ring with the property that $a^2 = a$ for all $a \in R$. Prove that any Boolean ring is commutative.

5. Consider the set $S = \left\{ \begin{bmatrix} a & c \\ c & b \end{bmatrix} : a, b, c \in \mathbb{Z} \text{ and } c = a - b \right\}$. Prove or disprove that S is a subring of $M_{22}(\mathbb{Z})$.

6. Consider the set $S = \{(a, b, c) : a, b, c \in \mathbb{Z} \text{ and } c = a + b\}$. Prove or disprove that S is a subring of $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.