Math 312- HW2 Due Date: 19/10/17

1. Let R be a ring with unity and G a finite nontrivial group. For any $g \in G$ with $g \neq e$, prove that $g - 1$ is a zero divisor in RG.

2. Let R be a ring with unity and $G = \{g_1, g_2, ..., g_n\}$ a finite group. Prove that the element $x = g_1 + g_2 + ... + g_n$ is in the center of RG.

3. Find all the units and zero divisors of $\mathbb{Z}_6 \times \mathbb{Z}_4$.

4. Find the characteristics of the following rings:

   (a) $\mathbb{Z}_4 \times \mathbb{Z}_6$ (b) $\mathbb{Z}_4 \times 4\mathbb{Z}$ (c) $M_{22}(\mathbb{Z}_5)$

   (d) The group ring RG where $R = \mathbb{Z}_4$ and $G = \mathbb{Z}_8$

5. An element a of a ring is an idempotent if $a^2 = a$. Prove that the only idempotents of an integral domain are 0 and 1.

6. In a commutative ring of characteristic 2, prove that the idempotents forms a subring