Math 312- HW3 Due Date: 31/10/17

1. Find a subring of $\mathbb{Z} \times \mathbb{Z}$ that is not an ideal of $\mathbb{Z} \times \mathbb{Z}$.
2. In the ring $\mathbb{Z}$, find a positive integer a such that
   (a) $\langle a \rangle = \langle 2 \rangle + \langle 3 \rangle$
   (b) $\langle a \rangle = \langle 3 \rangle + \langle 6 \rangle$
   (c) $\langle a \rangle = \langle 6 \rangle + \langle 8 \rangle$
3. If I and J are ideals in a commutative ring with unity and $I + J = R$, show that $IJ = I \cap J$.
4. Let $R = \mathbb{Z}_8 \times \mathbb{Z}_{30}$. Find all maximal ideals of R, and for each maximal ideal I, identify the size of the field R/I.
5. In $\mathbb{Z} \times \mathbb{Z}$, let $I = \{(a, 0) \mid a \in \mathbb{Z}\}$. Show that I is a prime ideal but not a maximal ideal.
6. Consider the set $S = \{(2a, 2b) \mid a, b \in \mathbb{Z}\}$. Prove or disprove that S is an ideal of $\mathbb{Z} \times \mathbb{Z}$.