Math 312- HW4 Due Date: 14/11/17

1. Let $\phi : R \to S$ be a surjective ring homomorphism and suppose that A is an ideal of R. Prove that $\phi(A) = \{\phi(a) \mid a \in A\}$ is an ideal of S.

2. Let $\phi : R \to S$ be a ring homomorphism and suppose that B is an ideal of S. Prove that $\phi^{-1}(B) = \{r \in R \mid \phi(r) \in B\}$ is an ideal of R.

3. Let R and S be nonzero rings with unity and denote their respective unities by 1_R and 1_S. Let $\phi : R \to S$ be a nonzero homomorphism of rings. Prove that if $\phi(1_R) \neq 1_S$ then $\phi(1_R)$ is a zero divisor in S. Deduce that if S is an integral domain then every ring homomorphism from R to S send the unity of R to the unity of S.

4. Let I be an ideal of a ring R and S a subring of R. Prove that $I \cap S$ is an ideal of S.

5. Assume that R is a commutative ring. Prove that if P is a prime ideal of R and P contains no zero divisors then R is an integral domain.

6. Assume that R is a commutative ring. Let I and J be ideals of R and assume that P is a prime ideal of R that contains IJ. Prove that either I or J is contained in P.