Math 312- HW5 Due Date: 28/11/17

1. Let R and S be commutative rings with unity. If ϕ is a surjective homomorphism from R to S and the characteristic of R is nonzero, prove that the characteristic of S divides the characteristic of R.

2. Prove that the rings R_1 and R_2 are not isomorphic

   (a) $R_1 = \mathbb{Q}, R_2 = \mathbb{Z}$ $\quad R_1 = \mathbb{Z}, R_2 = 2\mathbb{Z}$ $\quad$ (b) $R_1 = M_{22}(\mathbb{Z}_4), R_2 = M_{22}(\mathbb{Z}_6)$
   (c) $R_1 = \mathbb{Z}, R_2 = \mathbb{Z} \times \mathbb{Z}$

3. Let R and S be nonzero commutative rings with unity and let $\phi : R \to S$ be a surjective ring homomorphism. Prove that S is an integral domain if and only if $\ker \phi$ is a prime ideal of R. Also prove that S is a field if and only if $\ker \phi$ is a maximal ideal of R.

4. Let R and S be rings. Consider the map $\phi : R \times S \to R$ given by $\phi((a, b)) = a$.

   (a) Prove ϕ is a ring homomorphism.

   (b) Prove that ϕ is surjective

   (c) Find $\ker \phi$

5. Assume that R is a nonzero ring with unity and S is a nonzero ring. If $\phi : R \to S$ is a surjective ring homomorphism, prove that $\phi(u)$ is a unit in S for every unit u in R.

6. Assume that $\phi : R \to S$ is a ring homomorphism. Assume that I is an ideal of R and J an ideal of S such that $\phi(I) \subseteq J$. Prove that the map $\psi : R/I \to S/J$ given by $\psi(a + I) = \phi(a) + J$ is a ring homomorphism. (Note: You must first show that ψ is well-defined)