1. Determine whether the element is irreducible in the given integral domain

   (a) 5 in $\mathbb{Z}$ (b) 12 in $\mathbb{Z}$ (c) $3x + 9$ in $\mathbb{Z}[x]$ (d) $3x + 9$ in $\mathbb{Q}[x]$

2. Let R be a ring and I an ideal of R. Let $I[x]$ be the ideal of $R[x]$ consisting of all polynomials with coefficients in I. Prove that $R[x]/I[x]$ is ring-isomorphic to $(R/I)[x]$

3. Use the previous problem to prove that the set of all polynomials with even integer coefficients is a prime ideal of $\mathbb{Z}[x]$

4. Find a polynomial of degree greater than zero in $\mathbb{Z}_4[x]$ that is a unit. Prove your result.

5. Suppose that a and b belong to an integral domain, $b \neq 0$ and a is not a unit. Show that $\langle ab \rangle$ is a proper subset of $\langle b \rangle$.

6. Let $R = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \;\middle|\; a, b \in \mathbb{Z} \right\}$. Prove that R is ring-isomorphic to $\mathbb{Z} \times \mathbb{Z}$.