1. Prove that each of the following polynomials is irreducible in $\mathbb{Z}[x]$
   (a) $p(x) = x^4 + 12x^2 - 18x + 24$
   (b) $p(x) = 4x^3 + 15x^2 - 60x + 180$
   (c) $p(x) = 2x^5 + 25x^4 + 15x^3 + 30$
   (d) $p(x) = x^6 + x^3 + 1$ (Hint: evaluate $p(x+1)$)
2. Prove that the polynomial $p(x) = x^3 + x + 1$ is irreducible in $R = \mathbb{Z}/2\mathbb{Z}[x]$. Deduce that $R/\langle p(x) \rangle$ is a field and determine the size of this field.
3. Find the irreducible factors of $x^8 - 1$ in $\mathbb{Q}[x]$
4. Find all monic irreducible polynomials of degree 2 in $\mathbb{Z}/3\mathbb{Z}[x]$.
5. Find an infinite set of integers n such that the polynomial $p(x) = x^5 + 20x + n$ is irreducible in $\mathbb{Z}[x]$
6. Let I be the smallest ideal in $\mathbb{Z}[x]$ that contains 2 and x. Prove that I is not a principal ideal.